# MELTING RATES OF VARIOUS SIZE 

HAILSTONES IN A LARGE VERTICAL WIND TUNNEL

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## ABSTRACT

Artificially made, spherical hailstones of radius $0.50-1.75 \mathrm{~cm}$ were suspended in the updraft of a large vertical wind tunnel, and allowed to melt for time intervals of one to five minutes. Until now, there have been no experimental results concerning melting of large and medium sized hail.

Calculations of mass and radius melting rates were made for a one minute time interval using the equations suggested by Macklin (1963); however, the value of $x$, the heat transfer coefficient, was recomputed. Agreement between experimental and theoretical values was within five percent.

In addition, melting profiles were calculated for hailstones falling from various heights. A threshold radius of 0.69 cm for stones falling from 5000 meters was found to agree well with the value of 0.73 cm calculated by Ludlam (1958) for stones falling from 5400 meters.

## INTRODUCTION

Mason (1956) has derived the basic melting rate equation for 3 mm diameter ice pellets at the interface of liquid paraffin and carbon tetrachloride; however, the expression is valid only if there is no shedding of meltwater. Both Blanchard (1957) and Macklin (1963) have found that water is shed from a melting hàilstone.

Ranz and Marshall (1953) have shown that for a ventilated hailstone of radius $r$, the heat transfer due to molecular conduction is given by,

$$
\begin{equation*}
h=\left(2.0+0.6 \operatorname{Re}^{1 / 2} \operatorname{Pr} 1 / 3\right) \mathrm{K} \Delta T / 2 r \tag{1}
\end{equation*}
$$

where $\operatorname{Pr}$ is the Prandtl number, $k$ the thermal conductivity of air, $\operatorname{Re}$ the Reynolds number and $\Delta T$ the temperature difference between the hailstone surface and the airstream. The heat transfer due to condensation or evaporation is given by,

$$
\begin{equation*}
h_{D}=\left(2.0+0.6 \operatorname{Re}^{1 / 2} S C^{1 / 3}\right) L V D \Delta \sigma / 2 r \tag{2}
\end{equation*}
$$

where $S c$ is the Schmidt number, $L_{V}$ the latent heat of vaporization, $D$ the diffusivity of water vapor in air, and $\Delta \sigma$ the difference in vapor density
between the hailstone surface and the airstream.
The total heat transfer, $H$, is simply the sum of $h$ and $h_{D}$.

$$
\begin{equation*}
H=\frac{2.0(K \Delta T+L \mathrm{LVD} \mathrm{\Delta} \mathrm{\sigma})+0.6 \mathrm{Re}^{1 / 2_{\beta}}}{2 r} \tag{3}
\end{equation*}
$$

where

$$
\begin{equation*}
\beta=\left(\mathrm{Pr}^{1 / 3_{\mathrm{K} \Delta \mathrm{~T}}+\mathrm{Sc}}{ }^{1 / 3} \mathrm{LvD} \mathrm{\Delta} \mathrm{\sigma}\right) \tag{4}
\end{equation*}
$$

Macklin (1963), however, in deriving equations for melting based on the heat transfer from falling stones, uses the following expression for the total heat transfer,

$$
\begin{equation*}
H=X \operatorname{Re}^{1 / 2_{\beta / 2} r} \tag{5}
\end{equation*}
$$

where $X$ is a numerical factor depending on the shape of the hailstone (Figure 1). He states that substitution of Eq. (5) for Eq. (3) does not result in any loss of accuracy. This is incorrect, as will be shown later.

Experimentally,

$$
\begin{equation*}
H=L f \Delta m / A^{t} \tag{6}
\end{equation*}
$$

where $\Delta m$ is the mass lost during time $t, L_{f}$ the latent heat of fusion, and A the surface area of the hailstone. The derived melting rate equations, in terms of mass and radius, are,

$$
\begin{equation*}
d m / d t=x \cdot \operatorname{ARe}^{1 / 2} \beta / 2 r L f \tag{7}
\end{equation*}
$$

and

$$
\begin{equation*}
r \mathrm{dr} / \mathrm{dt}=\mathrm{x} \gamma \mathrm{Re}^{1 / 2} \beta / 2 \alpha \mathrm{~L} \mathrm{f}_{\rho} \mathrm{i} \tag{8}
\end{equation*}
$$

where $\gamma=A / 4 \pi r^{2}, \alpha$ is the ratio of the minor to major axes of the hailstone and $p i$ is the density of ice. Substitution of $\rho i=0.92 \mathrm{~g} \mathrm{~cm}-3$ into Eq. (8) yields,

$$
\begin{equation*}
r_{0}^{7 / 4}-r_{f}^{7 / 4}=2.48 \times 10^{-3} \chi \gamma C_{D}{ }^{1 / 4}-5 / 4 \int_{0}^{20} p^{3 / 4} \beta_{\beta \mu}^{-1 / 2} d z \tag{9}
\end{equation*}
$$

where $c_{0}$ is the drag coefficient of the hailstone, $\rho$ is the air density and $\mu$ is the dynamic viscosity of air.

At this time, however, there are no experimental results which deal with the melting of medium and large sized hailstones. This research has attempted to fill that gap and verify the theoretical results obtained by Macklin (1963).

## EXPERIMENTAL PROCEDURE

A large vertical wind tunnel was constructed for the study of hydrometeors suspended in an updraft. Its vertical cylinder is 1.8 m in diameter and 3 m in length. To streamline the air flow, cross hatched plywood was placed in the base of the vertical section. Three metal screens were placed at distances of $0.3,1.0$ and 7.5 m , respectively, above the plywood hatching. At the top of the tunnel, an 8 cm thick hexagonal honeycomb was placed to


Figure 1: Relationship between $x$, the heat transfer coefficient, and $\alpha$, the major to minor axis ratio (after Macklin, 1963).
further reduce turbulence in the flow. More details of the design and construction of this tunnel have been described by Spengler and Gokhale (1970). The tunnel has been used in earlier research to study icicle lobe structure of a hailstone (Gokhale and Spengler, 1973).

The working area was equipped with a gram balance, wet and dry bulb thermometers, a cold chamber, a stopwatch, and a calibrated HastingsRaydist hot-wire anemometer. The hot-wire probe was secured slightly off center in the updraft to obtain a true velocity reading without interfering with the airflow around the hailstone.

The hailstones themselves were made in two different ways. The large stones, having radii of $1.25-1.75 \mathrm{~cm}$. were made by freezing tap water inside spherical molds. Barklie (1962) found that water obtained from melting hailstones has the same purity as tap water. The molds were made from table tennis balls sliced in half, then taped together. A small hole was cut in each mold so water could escape during expansion. The molds were filled to the top, thus insuring hailstones of spherical shape. The smaller hailstones were frozen in ice trays which produced spherical hailstones of radius approximately 1.05 cm . Smaller hailstones were made by allowing these original stones to melt slightly, then removing them from the molds and refreezing in the cold chamber. The temperature was kept constant at -20 C. A length of nylon thread was frozen into each hailstone and used to suspend it in the updraft.

The wind tunnel fan was capable of maintaining the desired updraft speed with a variation of only $\pm 0.05 \mathrm{~m} \mathrm{sec}-7$.

Wet and dry bulb thermometers were inserted into the updraft to measure the temperature and relative humidity of the airflow.

The temperature of the cold chamber was low enough to allow at least 30 seconds to a minute for the weighing to be done before any appreciable melting set in. However, it usually took only about 25 seconds or so to weigh the hailstone and hang it in the updraft. During this short period of time, no melting was observed.

The hailstones were suspended for time intervals of one to five minutes. These runs were repeated to check the consistency of the results. Calculations for $\mathrm{dm} / \mathrm{dt}$ and $\mathrm{dr} / \mathrm{dt}$ were made for a time inverval of one minute.

## EXPERIMENTAL RESULTS

Artificial hailstones were suspended in an updraft of $9.5 \mathrm{~m} \mathrm{sec}^{-1}$. The temperature was $+22^{\circ} \mathrm{C}$.

The values of $d r / d t$ and $d m / d t$ are shown in Figure 2, which gives averages over a one minute melting period. The dashed lines are theoretical curves while the plotted points have been experimentally obtained.

It must be pointed out that Macklin (1963) used a value of $H$ which did not correspond to the value obtained by summing $h$ and $h_{D}$. The heat transfer coefficient, $x$, introduced in Eq. (5), takes the value of 0.76 for spherical hailstones, according to Macklin. However, if Eq. (5) is to be used for $H$, then $x$ must be adjusted to take into account the discrepancy between values of $H$ obtained from Eq.(5) and from Eq. (3). This adjusted value of $x$, calculated to be 0.62 , gives good agreement with experimental values when used in Eq. (5).

Experimental values of $\mathrm{dm} / \mathrm{dt}$ ranged between $9.4 \times 10^{-3} \mathrm{~g} \mathrm{sec}{ }^{-1}$ and 4.5 $\times 10^{-2} \mathrm{~g} \mathrm{sec}{ }^{-1}$. Values of $\mathrm{dr} / \mathrm{dt}$ ranged from $1.4 \times 10^{-3} \mathrm{~cm} \mathrm{sec}-1$ and 2.4 x $10^{-3} \mathrm{~cm} \mathrm{sec}-1$. All the experimental values found for both $\mathrm{dm} / \mathrm{dt}$ and $\mathrm{dr} / \mathrm{dt}$ were within $5 \%$ of the values calculated from the theoretical equations.

## CALCULATIONS OF HAILSTONE MELTING

Assuming that Macklin's equations for hailstone melting are essentially correct, the actual melting profile of a falling hailstone has been calculated. Note that the experimentally derived value of $x=0.62$ has been used throughout the calculations.

The fallspeed of the hailstones was determined through use of the relationship,

$$
\begin{equation*}
v^{2}=8 g \rho_{i r} / 3 p c_{0} \tag{10}
\end{equation*}
$$

where $g$ is the acceleration of gravity and all other variables have their


Figure 2: Melting rates of hailstones suspended in an updraft of $T=22^{\circ} \mathrm{C}$ and $V=9.5 \mathrm{~m} \mathrm{sec}-1$.


Figure 3: Melting profiles of various size hailstones falling from $0^{\circ} \mathrm{C}$ isotherm at $3 \mathrm{~km}, 4 \mathrm{~km}$ and 5 km .
previously defined meaning. The value of $C_{D}$, the drag coefficient for spherical hailstones, was calculated by Macklin and Ludlam (1961) and found to be approximately 0.45 .

The equation used to establish the hailstone radius after each 1000 meters of fall was,

$$
\begin{equation*}
\gamma f^{3 / 2}=\gamma_{0}^{3 / 2}-(2 \rho V / \mu)^{1 / 2} \beta z / 2 V L_{f} \rho i \tag{11}
\end{equation*}
$$

Calculations were made for three different hailstone sizes falling from
three different $0^{\circ} \mathrm{C}$ isotherm heights. The results have been plotted in Figure 3 as a function of fall time. Extrapolation to zero radius gives melting times which are in considerable disagreement with those calculated by Drake and Mason (1966). They calculated that hailstones with diameters of $1.0,2.0$, and 4.0 cm would melt completely in 11, 35, and 105 seconds, respectively, if suspended in an updraft of $3.5 \mathrm{~m} \mathrm{sec}^{-1}$ with a temperature of $+10^{\circ} \mathrm{C}$.

It was calculated that hailstones with radius less than 0.69 cm would melt before reaching the ground when falling from 00 C isotherm at 5000 m . This value compares favorably with the threshold radius value of 0.73 cm calculated by Ludlam (1958) for hailstones falling from 00 C isotherm at 5400 m .

## DISCUSSION AND CONCLUSIONS

The results of this experimental research tend to verify the equations given by Macklin (1963) for the melting of hailstones; however, some slight modification is suggested. The value of 0.76 for $x$, the heat transfer coefficient, given by Macklin, is apparently incorrect. An experimentally derived value of 0.62 gives much better agreement between theoretical and experimental values.

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