"REVIEWED"

A NEW NUMERICAL SIMULATION TECHNIQUE FOR WEATHER MODIFICATION EXPERIMENTS

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Abstract. A new numerical simulation technique for cloud seeding experiments is developed. The technique involves application of ratio estimators and requires coefficients of rainfall variation and correlation of rainfall of the target and control areas as input data. Results of the simulation experiments for the Indian region and the merits of the new technique are described.

1. INTRODUCTION

The probability of detecting prescribed increases in rainfall due to seeding with a specified degree of confidence can be determined by the numerical simulation of cloud seeding experiments (EXP-TR) developed by Twomey and Robertson (1973). These computer simulation experiments involve collection of long period historic rainfall data and considerable computer time even on high-speed modern computers. A different numerical simulation technique (EXP-MMM-I) has been developed and tested by Mary Selvam et. al. 1979. This technique not only reduces the computational time by an order of magnitude but also defines the exact lower limit for the double ratio value which can be detected at 5 percent level of significance.

In the present paper the authors have developed a new simulation technique (EXP-MMM-II) involving application of ratio estimators. This technique requires coefficients of rainfall variability and correlations of rainfall of the target and control areas as input data for the computer simulation of cloud seeding experiments. Such experiments were carried out and the probability of detection of prescribed increases in rainfall due to seeding for the experiments with (i) double-area cross-over design and area randomization and (ii) the fixed target-control area design and day randomization. For these experiments coefficients of rainfall variation computed from the weekly total rainfall of the 35 meteorological subdivisions in India for the 5-year period (1976-80) were used. Also, numerical simulation of cloud seeding experiments using the historic rainfall data of a few regions in India, and the results of EXP-MMM-II and EXP-TR are compared. The theory relating to the new numerical simulation technique (EXP-MM-II), results of comparison between EXP-TR and EXP-MMM-II, nomograms for detection of prescribed increases in rainfall due to seeding and a map showing probabilities of detecting prescribed increases in rainfall due to seeding in different regions in India are presented below.

2. EXPERIMENTAL DESIGNS

2.1 Double-area cross-over design

In this experiment two areas A and B are separated by a buffer area and cloud seeding experiment is carried out in one of the areas on a random basis. The change in rainfall due to seeding is estimated by the root double ratio, R_S:

$$R_S = \left[\frac{\Sigma A_S}{\Sigma A_{NS}} \times \frac{\Sigma B_S}{\Sigma B_{NS}}\right]^{1/2} \qquad \ldots(1)$$

where the subscript S refers to the seeded rainfall and NS to the not-seeded rainfall for the areas A and B.

For an experiment with the duration of N days, each of the areas A and B will be seeded on N/2 days and this number is designated as n.

2.2 Fixed target-control area design

In this experiment the target area is seeded on random basis on certain days. The change in rainfall due to seeding is estimated by the double ratio, r_S:

$$r_S = \left[\frac{\Sigma A_S}{\Sigma B_S} \Big/ \frac{\Sigma A_{NS}}{\Sigma B_{NS}}\right]$$

$$= \left[\frac{\Sigma A_S}{\Sigma A_{NS}} \times \frac{\Sigma B_{NS}}{\Sigma B_S}\right] \qquad \ldots(2)$$

where S refers to the seeded rainfall and NS to the not-seeded rainfall for the two areas A and B.

For an experiment with the duration of N days, which is long enough, the number of seeded days n will be almost equal to N/2.

In the case of the experiment with fixed target-control area design, A_S and B_S relate to the same day. Similarly in the case of the experiment with double-area cross-over design (Section 2.1) A_S and B_{NS} also relate to the same day. Hence for the same historic rainfall data set, R_S^2 (Equation 1) and r_S (Equation 2) will be the same.

3. DISTRIBUTION OF DOUBLE RATIO/ROOT DOUBLE RATIO

Flueck and Holland (1976) studied some of the properties (mean and standard deviation) of ratio estimators and used the results of their study for the re-evaluation of three well known cloud seeding experiments. The mean, standard deviation, skewness and kurtosis of the double ratio and root double ratio distributions relating to the cloud seeding experiments are derived in the following using the ratio method of estimation (Sukhatme and Sukhatme 1970).

3.1 Experiment with double-area cross-over design

Let $A_1, A_2 \ldots\ldots A_n$ and $B_1, B_2 \ldots\ldots B_n$ denote the historic daily rainfall data of the areas A and B respectively for an experiment with the duration of N days.

The double ratio r is expressed as :

$$r = \left[\frac{(\sum_1^n A_i)_S}{(\sum_1^n A_i)_{NS}} \times \frac{(\sum_1^n B_i)_S}{(\sum_1^n B_i)_{NS}} \right]$$

$$= \left[\frac{\bar{A}_S}{\bar{A}_{NS}} \times \frac{\bar{B}_S}{\bar{B}_{NS}} \right] \qquad \ldots(3)$$

The mean rainfall of the two areas is indicated by $\bar{A}$ and $\bar{B}$. In the above experiment there are N_{C_n} possible ways of choosing n seeded days out of the total N days. Hence $\bar{A}_S$, $\bar{A}_{NS}$, $\bar{B}_{NS}$ are distributed normally, since they come from a very large population of N_{C_n} values.

Let ρ be the correlation coefficient (which is generally significant at 5% level) between the rainfall of the two areas A and B, S_A and S_B the standard deviations, C_A and C_B the coefficients of variation in rainfall of the two areas A and B.

As there is good correlation between the rainfall of the two areas A and B the mean rainfall $(\bar{A}_S)$ of seeded days of the area A, can be expressed as

$$\bar{A}_S = \bar{A}_N \pm \bar{\varepsilon}_n' \qquad \ldots(4)$$

where $\bar{A}_N$ is the mean rainfall of the total experimental days and $\bar{\varepsilon}_n'$ is a small variation in rainfall from $\bar{A}_N$. Similarly the mean rainfall $(\bar{B}_S)$ of seeded days of the area B can be expressed

$$\bar{B}_S = \bar{B}_N \mp \bar{\varepsilon}_n \qquad \ldots(5)$$

In case of not-seeded days the above equations can be expressed as

$$\bar{A}_{NS} = \bar{A}_N \mp \bar{\varepsilon}_n' \qquad \ldots(6)$$

$$\bar{B}_{NS} = \bar{B}_N \pm \bar{\varepsilon}_n' \qquad \ldots(7)$$

In the above equations the magnitude of $\bar{\varepsilon}_n'$ and and $\bar{\varepsilon}_n$ are very small when compared to the mean rainfall i.e. $\bar{\varepsilon}_n' / \bar{A}_N << 1$ and $\bar{\varepsilon}_n/\bar{B}_N << 1$.

3.2 Experiment with fixed target-control area design

In this experiment the seeded day for the target area (A) will be the not-seeded day for the control area (B). Hence expressions for $\bar{A}_S$, $\bar{B}_{NS}$, $\bar{B}_S$ and $\bar{A}_{NS}$ (Equations 4 to 7) are valid for the experiment with fixed target-control area design. The expected values of $\bar{\varepsilon}_n'$, $\bar{\varepsilon}_n$, $\bar{\varepsilon}_n'^2$, $\bar{\varepsilon}_n^2$ & $\overline{\varepsilon_n' \varepsilon_n}$ can be expressed (Sukhatme and Sukhatme, 1970) in the following form.

$$E(\bar{\varepsilon}_n) = 0$$

$$E(\bar{\varepsilon}_n^2) = \frac{N-n}{N\,n} S_B^2 = \frac{S_B^2}{N} = S_b^2$$

$$= \text{Variance of } \bar{B}_S(\bar{B}_{NS}) \qquad \ldots(8)$$

$$E(\bar{\varepsilon}_n') = 0$$

$$E(\bar{\varepsilon}_n'^2) = \frac{N-n}{N\,n} S_A^2 = \frac{S_A^2}{N} = S_a^2$$

$$= \text{Variance of } \bar{A}_S(\bar{A}_{NS}) \qquad \ldots(9)$$

$$E(\overline{\varepsilon_n \varepsilon_n'}) = \frac{\rho S_B S_A}{N}$$

$= \text{Cross-covariance between } \bar{A}_S, \bar{B}_S$ and between $\bar{A}_{NS}, \bar{B}_{NS} \qquad \ldots(10)$

3.2.1 Mean of the double ratio estimator, r.

The mean of the double ratio estimator r, can be expressed as follows:

$$r = \frac{(\bar{A}_N \pm \bar{\varepsilon}_n')}{(\bar{A}_N \mp \bar{\varepsilon}_n')} \times \frac{(\bar{B}_N \mp \bar{\varepsilon}_n)}{(\bar{B}_N \pm \bar{\varepsilon}_n)}$$

$$r = \left[1 \pm \frac{\bar{\varepsilon}_n'}{\bar{A}_N}\right] \left[1 \mp \frac{\bar{\varepsilon}_n'}{\bar{A}_N}\right]^{-1} \left[1 \mp \frac{\bar{\varepsilon}_n}{\bar{B}_N}\right] \left[1 \pm \frac{\bar{\varepsilon}_n}{\bar{B}_N}\right]^{-1} \qquad (11)$$

Expanding the terms involving negative powers of one by Taylor's theorem and neglecting terms higher than $\left[\frac{\bar{\varepsilon}_n'}{\bar{A}_N}\right]^2$ and $\left[\frac{\bar{\varepsilon}_n}{\bar{B}_N}\right]^2$,

the following expression for, r, is obtained

$$r = 1 \pm 2 \left[\frac{\bar{\varepsilon}_n}{\bar{B}_N} - \frac{\bar{\varepsilon}_n'}{\bar{A}_N} \right] + 2 \left[\frac{\bar{\varepsilon}_n}{\bar{B}_N} - \frac{\bar{\varepsilon}_n'}{\bar{A}_N} \right]^2 \qquad \ldots(12)$$

The expected value of r, i.e., E(r) can be expressed as follows:

$$E(r) = 1 \pm 2\, E\left[\frac{\bar{\varepsilon}_n}{\bar{B}_N} - \frac{\bar{\varepsilon}'_n}{\bar{A}_N}\right] + 2\, E\left[\frac{\bar{\varepsilon}_n}{\bar{B}_N} - \frac{\bar{\varepsilon}'_n}{\bar{A}_N}\right]^2$$

$$= 1 + \frac{2}{N}\left[C_A^2 + C_B^2 - 2\,\rho C_A\, C_B\right] \quad \ldots(13)$$

3.2.2 Variance of the double ratio estimator, r.

The variance of r is given by the second moment (m_2) about the mean and expressed as follows:

$$m_2 = E\left[r - E(r)\right]^2$$

$$= E\left[\pm 2\left(\frac{\bar{\varepsilon}_n}{\bar{B}_N} - \frac{\bar{\varepsilon}'_n}{\bar{A}_N}\right) + 2\left(\frac{\bar{\varepsilon}_n}{\bar{B}_N} - \frac{\bar{\varepsilon}'_n}{\bar{A}_N}\right)^2\right]^2$$

$$m_2 \cong 4\, E\left[\frac{\bar{\varepsilon}_n}{\bar{B}_N} - \frac{\bar{\varepsilon}'_n}{\bar{A}_N}\right]^2 \pm 8\, E\left[\frac{\bar{\varepsilon}_n}{\bar{B}_N} - \frac{\bar{\varepsilon}'_n}{\bar{A}_N}\right]^3 \quad (14)$$

Since $\bar{A}_N$, $\bar{A}_{NS}$, $\bar{B}_N$, $\bar{B}_{NS}$ are normally distributed, the second term on the R.H.S. of Equation (14), i.e.,

$$E\left[\frac{\bar{\varepsilon}_n}{\bar{B}_N} - \frac{\bar{\varepsilon}'_n}{\bar{A}_N}\right]^3 = 0 \text{ as shown below.}$$

$$\left[\frac{\bar{\varepsilon}_n}{\bar{B}_N} - \frac{\bar{\varepsilon}'_n}{\bar{A}_N}\right]^3$$

$$= \frac{\bar{\varepsilon}_n^{\,3}}{\bar{B}_N^{\,3}} - \frac{3\,\bar{\varepsilon}_n^{\,2}\,\bar{\varepsilon}'_n}{\bar{B}_N^{\,2}\,\bar{A}_N} + \frac{3\,\bar{\varepsilon}_n\,\bar{\varepsilon}'^{\,2}_n}{\bar{B}_N\,\bar{A}_N^{\,2}} - \frac{\bar{\varepsilon}'^{\,3}_n}{\bar{A}_N^{\,3}} \quad \ldots(15)$$

Writing the above in terms of the moment notation given by

$$\mu_{\alpha,\,\alpha} = \frac{1}{N}\sum_{i=1}^{N} \varepsilon_i^{\alpha}\, \varepsilon_i'^{\alpha}$$

the expected values of the product moments (Sukhatme and Sukhatme, 1970) are obtained.

$$\mu_{r+s} = 0 \quad \text{if } (r+s) \text{ is odd} \quad \ldots(16)$$

Hence

$$E\left[\frac{\bar{\varepsilon}_n}{\bar{B}_N} - \frac{\bar{\varepsilon}'_n}{\bar{A}_N}\right]^3 = 0 \quad \ldots(17)$$

Therefore the second moment (m_2) about the mean for the r distribution i.e., the variance can be expressed as follows.

$$m_2 = 4\, E\left[\frac{\bar{\varepsilon}_n}{\bar{B}_N} - \frac{\bar{\varepsilon}'_n}{\bar{A}_N}\right]^2$$

$$= 4\, E\left[\frac{\bar{\varepsilon}_n^{\,2}}{\bar{B}_N^{\,2}} \quad \frac{\bar{\varepsilon}'^{\,2}_n}{\bar{A}_N^{\,2}} - 2\,\frac{\bar{\varepsilon}_n\,\bar{\varepsilon}'_n}{\bar{B}_N\,\bar{A}_N}\right]$$

$$m_2 = \frac{4}{N}\left(C_A^2 + C_B^2 - 2\,\rho\, C_A\, C_B\right) \quad \ldots(18)$$

3.2.3 Skewness of the distribution of r

The moment coefficient of skewness $= \dfrac{m_3}{\sqrt{m_2^{\,3}}}$ ...(19)

where m_3 is the third moment about the mean for the r distribution and can be expressed as follows:

$$m_3 = E\left[r - E(r)\right]^3$$

$$= E\left[\pm 2\left(\frac{\bar{\varepsilon}_n}{\bar{B}_N} - \frac{\bar{\varepsilon}'_n}{\bar{A}_N}\right) + 2\left(\frac{\bar{\varepsilon}_n}{\bar{B}_N} - \frac{\bar{\varepsilon}'_n}{\bar{A}_N}\right)^2\right]^3 \quad \ldots(20)$$

Neglecting powers of $\left[\frac{\bar{\varepsilon}_n}{\bar{B}_N} - \frac{\bar{\varepsilon}'_n}{\bar{A}_N}\right]$ higher than 4, Equation (20) can be written as

$$m_3 = E\left[\pm 8\left(\frac{\bar{\varepsilon}_n}{\bar{B}_N} - \frac{\bar{\varepsilon}'_n}{\bar{A}_N}\right)^3 \pm 24\left(\frac{\bar{\varepsilon}_n}{\bar{B}_N} - \frac{\bar{\varepsilon}'_n}{\bar{A}_n}\right)^4\right]$$

$$= 24\, E\left[\left(\frac{\bar{\varepsilon}_n}{\bar{B}_N} - \frac{\bar{\varepsilon}'_n}{\bar{A}_n}\right)^4\right]$$

Since the first term $E\left[\frac{\bar{\varepsilon}_n}{\bar{B}_N} - \frac{\bar{\varepsilon}'_n}{\bar{A}_N}\right]^3$ has been shown to be equal to zero for bivariate normal distribution.

$$m_3 = \pm 24\, E\left[\frac{\bar{\varepsilon}_n^4}{\bar{B}_N^4} - 4\frac{\bar{\varepsilon}_n^3\,\bar{\varepsilon}_n'}{\bar{B}_N^3\,\bar{A}_N} + 6\frac{\bar{\varepsilon}_n^2\,\bar{\varepsilon}_n'^2}{\bar{B}_N^2\,\bar{A}_N^2} - 4\frac{\bar{\varepsilon}_n\,\bar{\varepsilon}_n'^3}{\bar{B}_N\,\bar{A}_N^3} + \frac{\bar{\varepsilon}_n'^4}{\bar{A}_N^4}\right] \tag{21}$$

The expected values of the product moments (Sukhatme and Sukhatme, 1970) are given as follows:

$$E\left[\bar{\varepsilon}_n^4\right] = \frac{3}{N^2}\, S_B^4 \tag{22}$$

Similarly for large values of N the approximate expressions for the higher product moments are written as follows:

$$E\left[\bar{\varepsilon}_n^3\;\bar{\varepsilon}_n'\right] = \frac{3}{N^2}\,\rho\, S_B^3\, S_A$$

$$E\left[\bar{\varepsilon}_n\;\bar{\varepsilon}_n'^3\right] = \frac{3}{N^2}\,\rho\, S_B\, S_A^3$$

$$E\left[\bar{\varepsilon}^2\;\bar{\varepsilon}'^2\right] = \frac{1}{N^2}\left[1 + 2\rho^2\right] S_B^2\; S_A^2$$

and $E\left[\bar{\varepsilon}_n'^4\right] = \frac{3}{N^2}\, S_A^4$

Hence $E\left[\frac{\bar{\varepsilon}_n}{\bar{B}_N} - \frac{\bar{\varepsilon}_n'}{\bar{A}_N}\right]^4$

$$= \frac{3}{N^2}\left[C_A + C_B - 2\rho\, C_A\, C_B\right]^2 \tag{23}$$

and $m_3 \approx \pm \frac{72}{N^2}\left[C_A^2 + C_B^2 - 2\rho\, C_A\, C_B\right]^2$

$$= \pm\frac{72}{16}\, m_2^2 = \frac{9}{2}\, m_2^2 \tag{24}$$

Moment coefficient of Skewness $= \pm\frac{72}{16}\,\frac{m_2^2}{m_2^{3/2}}$

$$= \pm\frac{9}{2}\sqrt{m_2}$$

$$= \pm\frac{9}{2}\sqrt{\text{Variance}} \tag{25}$$

The standard error of skewness $= \sqrt{6/n}$

Hence the r distribution is symmetrical if,

$$= \pm\frac{9}{2}\sqrt{m_2} \leqslant 2\sqrt{6/n}$$

or Variance $\leqslant 1.182/N$

3.2.4 Kurtosis of the distribution of r

The moment coefficient of Kurtosis $= m_4 / m_2^2$ (26)

where m_4 is the fourth moment about the mean for the r distribution and can be expressed as

$$= E\left[r - E(r)\right]^4 \tag{27}$$

$$= E\left[\pm 2\left(\frac{\bar{\varepsilon}_n}{\bar{B}_N} - \frac{\bar{\varepsilon}_n'}{\bar{A}_N}\right) + 2\left(\frac{\bar{\varepsilon}_n}{\bar{B}_N} - \frac{\bar{\varepsilon}_n'}{\bar{A}_N}\right)^2\right]^4$$

$$\approx E\left[16\left(\frac{\bar{\varepsilon}_n}{\bar{B}_N} - \frac{\bar{\varepsilon}_n'}{\bar{A}_N}\right)^4 \pm 64\left(\frac{\bar{\varepsilon}_n}{\bar{B}_N} - \frac{\bar{\varepsilon}_n'}{\bar{A}_N}\right)^5\right]$$

$$= 16\, E\left[\frac{\bar{\varepsilon}_n}{\bar{B}_N} - \frac{\bar{\varepsilon}_n'}{\bar{A}_N}\right]^4$$

$$\approx \frac{48}{N^2}\left[C_A^2 + C_B^2 - 2\rho\, C_A\, C_B\right]^2 = 3\, m_2^2 \tag{28}$$

Hence the moment coefficient of Kurtosis = 3

i.e. Kurtosis of the r distribution is the same as that for the normal distribution.

The skewness is however positive for the distribution when $r > 1$ and negative for distribution when $r < 1$.

The double ratio estimator r is used to evaluate the seeding effect in the experiment with fixed target-control area design.

3.3 Experiment with double-area cross over design

3.3.1 Mean of the root double ratio estimator R

The mean of the root double ratio estimator R can be expressed as follows:

$$R = \sqrt{r} \tag{29}$$

Using equation (12) the above expression can be written as

$$R = \left[1 \pm 2\left(\frac{\bar{\varepsilon}_n}{\bar{B}_N} - \frac{\bar{\varepsilon}_n'}{\bar{A}_N}\right) + 2\left(\frac{\bar{\varepsilon}_n}{\bar{B}_N} - \frac{\bar{\varepsilon}_n'}{\bar{A}_N}\right)^2\right]^{1/2} \tag{30}$$

$$\approx 1 + \frac{1}{2}\left[\pm 2\left(\frac{\bar{\varepsilon}_n}{\bar{B}_N} - \frac{\bar{\varepsilon}'_n}{\bar{A}_N}\right) + 2\left(\frac{\bar{\varepsilon}_n}{\bar{B}_N} - \frac{\bar{\varepsilon}'_n}{\bar{A}_N}\right)^2\right] - \frac{1}{8}\left[\pm 2\left(\frac{\bar{\varepsilon}_n}{\bar{B}_N} - \frac{\bar{\varepsilon}'_n}{\bar{A}_N}\right) + 2\left(\frac{\bar{\varepsilon}_n}{\bar{B}_N} - \frac{\bar{\varepsilon}'_n}{\bar{A}_N}\right)^2\right]^2$$

$$R \approx 1 \pm \left(\frac{\bar{\varepsilon}_n}{\bar{B}_N} - \frac{\bar{\varepsilon}'_n}{\bar{A}_N}\right) + \frac{1}{2}\left(\frac{\bar{\varepsilon}_n}{\bar{B}_N} - \frac{\bar{\varepsilon}'_n}{\bar{A}_N}\right)^2 \mp \left(\frac{\bar{\varepsilon}_n}{\bar{B}_N} - \frac{\bar{\varepsilon}'_n}{\bar{A}_N}\right)^3 - \frac{1}{2}\left(\frac{\bar{\varepsilon}_n}{\bar{B}_N} - \frac{\bar{\varepsilon}'_n}{\bar{A}_N}\right)^4 \quad (31)$$

The expected value of R, i.e., E(R) can be expressed as follows:

$$E(R) \approx 1 + \frac{1}{2N}\left[C_A^2 + C_B^2 - 2\rho C_A C_B\right] \quad (32)$$

3.3.2 Variance of the root double ratio estimator, R.

The variance of R is given by the second moment (m_2) about the mean and can be expressed as follows:

$$m_2 = E\left[R - E(R)\right]^2 \quad (33)$$

$$m_2 \approx E\left(\frac{\bar{\varepsilon}_n}{\bar{B}_N} - \frac{\bar{\varepsilon}'_n}{\bar{A}_N}\right)^2 + E\left[\pm\left(\frac{\bar{\varepsilon}_n}{\bar{B}_N} - \frac{\bar{\varepsilon}'_n}{\bar{A}_N}\right)^3\right]$$

$$= \frac{1}{N}\left[C_A^2 + C_B^2 - 2\rho C_A C_B\right] \quad (34)$$

3.3.3 Skewness of the root double ratio estimator R.

$$\text{The moment coefficient of Skewness} = \frac{m_3}{\sqrt{m_2^3}} \quad (35)$$

where M_3 is the third moment about the mean for the root double ratio and can be expressed as follows:

$$m_3 = E\left[R - E(R)\right]^3$$

$$= E\left(\frac{\bar{\varepsilon}_n}{\bar{B}_N} - \frac{\bar{\varepsilon}'_n}{\bar{A}_N}\right)^3 + E\left[\pm\frac{3}{2}\left(\frac{\bar{\varepsilon}_n}{\bar{B}_N} - \frac{\bar{\varepsilon}'_n}{\bar{A}_N}\right)^4\right]$$

$$= \frac{9}{2N^2}\left[C_A^2 + C_B^2 - 2\rho C_A C_B\right]^2 = \frac{9}{2} m_2^2 \quad (37)$$

Hence the distribution of the root double ratio is symmetric about the mean if the variance is $< \frac{1.182}{N}$.

Since the variance of the root double ratio (R) is 1/4th of the variance for the double ratio (r) the skewness of the root double ratio is reduced by half as compared to that for the double ratio. For most of the cases considered in this study the skewness of the root double ratio distribution is less than twice the standard error for skewness and hence the root double ratio distributions are normal. However, from the computations it is found that when the coefficients of rainfall variability exceed 2.0, the correlation coefficient between the rainfall of the target and control areas should be more than 0.7 for the root double ratio to be normal.

3.3.4 Kurtosis of the root double ratio estimator R.

$$\text{The moment coefficient of Kurtosis} = \frac{m_4}{m_2^2}$$

where m_4 is the fourth moment about the mean for the R distribution and can be expressed as:

$$m_4 = E\left[R - E(R)\right]^4 \quad (38)$$

$$= E\left(\frac{\bar{\varepsilon}_n}{\bar{B}_N} - \frac{\bar{\varepsilon}'_n}{\bar{A}_N}\right)^4$$

$$= \frac{3}{N^2}\left[C_A^2 + C_B^2 - 2\rho C_A C_B\right]^2 = 3 m_2^2 \quad (39)$$

Hence the moment coefficient of Kurtosis for the R distribution $= \frac{m_4}{m_2^2} = 3$

Hence the Kurtosis for the distribution of R and for the normal distribution is the same. The root double ratio is thus proved to be distributed normally for large values of N.

3.4 Detection of significant root double ratio values

For the root double ratio values which are distributed normally the 5 percent level of significance can be evaluted as follows:

The standard deviation (S) of the root double ratio distribution is equal to $\sqrt{\text{variance}}$.

Let R_d denote the value of R (>1.0) which is significant at 5 percent level and can be expressed as

$$R_d = E(R) + 1.655\ S = R_m + 1.65\ S \qquad (40)$$

where R_m is the mean value of the root double ratio distribution and $R_m = E(R) \cong 1.0$

Hence the values of $R \geqslant R_d$ in the R distribution can be detected i.e., significant at $\leqslant$ 5% level of significance.

4. SIMULATION OF SEEDING EFFECT

Let the percentage increase in rainfall in the target area due to seeding be designated as PERC.

4.1 Double area cross-over design

The distribution of the root double ratio values with simulated seeding effect (R_s) can be derived starting from the R distribution as follows:

$$R_s = \left[1 + \frac{PERC}{100}\right] R \qquad (41)$$

The percentage probability of detection (P) of a simulated percentage increase PERC in the rainfall of the target area can be evaluated as follows:

Let P be equal to the percentage number of R_s values which are significant at the 5 percent level. The lower limit of detection is R_d for the root double ratio values greater than one.

All values of $R \geqslant \left[\frac{R_d}{1+\frac{PERC}{100}}\right]$ in the R distribution will have values greater than or equal to R_d in the R_s distribution.

$$\text{Let} \quad R_p = \left[\frac{R_d}{1 + \frac{PERC}{100}}\right] \qquad (42)$$

The percentage probability of detection (P) will thus be given by the percentage probability of occurrence of values of $R \geqslant R_p$ in the R distribution.

Since R is distributed normally the P value can be determined by computing the standardized normal deviate (Z) using the following expression.

$$Z = (R_p - R_m) / S \qquad (43)$$

The probability of occurrence of Z can be read from the tables of the standardized normal probability integrals (Fisher and Yates, 1974).

4.2 Fixed target-control area design

The percentage probability of detection (P) in the case of a fixed target-control design can be determined as follows:

In Section 2.2 it was shown that R^2 and r are equal. The distribution of the root double ratio values for fixed target-control design with simulated seeding effect (R_s) can be derived from the R distribution as follows:

$$R_s = \left[\sqrt{1 + \frac{PERC}{100}}\right] R \qquad (44)$$

All values of $R \leqslant \frac{R_d}{\sqrt{1+\frac{PERC}{100}}}$ in the distribution will have values greater than or equal to R_d in the r_s distribution where,

$$R_s = \left[\frac{R_d}{\sqrt{1 + \frac{PERC}{100}}}\right] \qquad (45)$$

5. ESTIMATION OF ERROR IN THE EVALUATION OF PROBABILITY OF DETECTION DUE TO LIMITED SAMPLE SIZE.

The error in the evaluation of the probability of detection (P) of seeding effect due to limited sample size can be determined as follows:

The standard error (C) of the standard deviation (S) is expressed as

$$C = \left[\frac{S}{\sqrt{2N}}\right] \qquad (46)$$

$$\text{Let} \quad f = \frac{C}{S} \qquad (47)$$

The range of values for the standard deviation will be $(S \pm C) = (S \pm fS)$. Hence the range of variation in R_d due to the standard error C will be:

$$[r_m + 1.65\ (S \pm fS)]$$

and from Equation (40) it can be expressed as....

$$[(R_d \pm 1.65\ fS)]$$

5.1 Double-area cross-over design

The range of variation R_p due to the standard error of the standard deviations can be written from Equation (42) as:

$$\left[\frac{R_d \pm 1.65\ fS}{(1 + \frac{PERC}{100})}\right]$$

Let dZ be the change in the value of the standardized normal deviate due to change in the R_p values:

$$dZ = \left[\frac{R_d}{1+\frac{PERC}{100}} - R_m\right]\frac{1}{S} - \left[\frac{R_d \pm 1.65\ fS}{1+\frac{PERC}{100}} - R_m\right]\frac{1}{S}$$

$$dZ = \pm\left[\frac{1.65\ f}{1+\frac{PERC}{100}}\right]$$

5.2 Fixed target-control design

$$dZ = \pm\frac{1.65\ f}{\sqrt{1+\frac{PERC}{100}}} \tag{49}$$

The deviation in the probability of detection (P) due to the standard error of the standard deviation can be computed as follows:

The standardized normal probability integral (p) is expressed as:

$$p = \frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}} \tag{50}$$

Taking logarithams and differentiating both sides of the above equation we get

$$\frac{dp}{p} = -ZdZ \tag{51}$$

Where $\frac{dp}{p}$ is the fractional change in probability due to a change (dZ) in the standardized normal deviate Z. Using the above method, variation in the probability of detection, dp, due to the error of the standard deviation can be computed for any set of P, Z and dZ values.

6. RESULTS AND CONCLUSIONS

Numerical simulation of cloud seeding experiments using the new technique (EXP-MMM-II) described in this paper were carried out for the Indian region and the results are given in Table 1. Also, results of the numerical simulation experiments (EXP-TR) carried out using the technique of Twomey and Robertson (1973) for north India (Delhi region) and for Maharashtra State (Mary Selvam et al., 1979, 1980, a,b) are given in Table 1 for comparison. The results of (EXP-MMM-II) and (EXP-TR) are in good agreement and the technique of (EXP-MMM-II) can be used for evaluating the probability of detection of prescribed increases in rainfall due to seeding in any region when data relating to the coefficients of rainfall variation and correlations are available. The error in the determination of the percentage probability of detection due to the limited sample size can also be evaluated as described in Section 5 of this paper. Also, the characteristics of the root double ratio distribution can be used to evaluate the significance of the results of the actual cloud seeding experiments as described in Section 3.

The nomograms for detection of prescribed increases in rainfall due to seeding are shown in Figure 1-3. The probabilities of detection of 10 and 15 percent increases in rainfall due to seeding were computed for different regions in India and maps showing the results are given in Figures 4 and 5 respectively.

In the numerical simulation experiments it is considered that for the successful detection of the seeding effect the probability of detection should be greater than 80 percent (Smith and Shaw, 1976). If this criteria is adopted the regions which are suitable for undertaking weather modification experiments in India during the summer monsoon (June-September) season are identified to be (i) northeast India, (ii) central India, (iii) west-coast and (iv) south India (Figure 5).

A cloud seeding experiment with double-area cross-over design and area randomization is in progress in the Pune region of Manharashtra state since 1973. The duration of the experiment required to detect the assigned percentage increases in rainfall due to seeding in the Pune region has been evaluated using the new numerical simulation technique (Section 4) and the results given in Table 2. These results indicate that increases in rainfall exceeding 10 percent can be detected sucessfully. The experiment should continue for a minimum period of 14 years for the detection of 10 percent increase in rainfall due to seeding.

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